Title: Talking with Machines--Binary Conversions

Brief Overview:

This lesson will model numerical values of machine language with that of other numbering systems. Students will use models to move from numerical systems to a machine system.

Links to NCTM Standards:

- **Mathematics as Problem Solving**
  Students will decode a secret message by converting systems to binary and decipher by using the ASCII codes.

- **Mathematics as Communication**
  Students will demonstrate their ability to communicate data through both oral and written techniques.

- **Mathematics as Reasoning**
  Students will demonstrate mathematical reasoning through application of binary data using varied number systems and machine codes.

- **Mathematical Connections**
  Students will relate mathematical numbering systems with current machine technology and computer programming codes.

- **Number Systems and Number Theory**
  Students will be able to convert octal, decimal, and hexadecimal to binary and vice versa.

- **Computation and Estimation**
  Students will be able to apply estimation values to solving hidden codes within computational guidelines.

- **Patterns and Functions**
  Students will recognize patterns of numbering systems through place value by position relationships of varied systems and codes.

- **Technology**
  Students will demonstrate their ability to use technology where appropriate as they solve real-world problems in machine communication.

Grade/Level:

Grades 6-12

Duration/Length:

2-3 days

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Working in the decimal system at a pre-algebra level
- Knowledge of exponents (powers)

Objectives:

Students will:

- convert binary, octal, and hexadecimal numbers to its decimal equivalent.
- convert decimal, octal, and hexadecimal numbers to its binary equivalent.

Materials/Resources/Printed Materials:

- Worksheets
- ASCII code chart

Development/Procedures:

Follow the detailed lesson plan. It is broken down into sections with step by step procedures.

Performance Assessment:

Students should be able to do the chalkboard examples at their desk after the provided example is given on the board. Upon completion of all sections the students should be able to do the provided worksheet that converts systems to binary. The ASCII code chart can be used to find the corresponding letter to decode the secret message (Math controls the world).

Extension/Follow Up:

Converting from Octal to hexadecimal, Binary to Octal, etc.
Truth tables
Boolean Algebra
Logic Gates
Arithmetic in Binary, Octal, etc.

Authors:

Jeremy Kurutz
Overlea High School
Baltimore County, MD

Jim Groves
Overlea High Schoo
Baltimore County, MD

Introductory Activity:

Purpose: A motivational puzzle that demonstrates the place value of digits in the decimal number system.

Find the number.

1. It is a three-digit whole number.
2. It is divisible by 5.
3. It is an even number.
4. Each of its digits is different.
5. Its tens digit is greater than its ones digit.
6. Its hundreds digit is greater than its tens digit.
7. It is less than 400.
8. It is divisible by three.
9. It has only one odd digit.

Answer: 210

Vocabulary:

Decimal number system
Binary number system
Octal number system
Hexadecimal number system
Digit
Base
Exponent
Place value
Bits
Assembly language

Digital integrated circuits
Transistor
Diode
Resistor
Boolean logic
Switching circuits

Decimal Number System

Decimal Number System -- A number system, having a base of 10. Digits are 0 through 9. It is the most popular system in use. Also referred to as the Arabic number system.

<u>Place Value by Position</u>

Digit name by position	Thousands	Hundreds	Tens	Ones
Exponential digit value by position	10^3	10^2	10^1	10^0

We can demonstrate the value of each position(*place value*) by analyzing a sample decimal number.

Example: 3210_{10}

$$
\begin{aligned}
3210_{10} &= 3 \times 10^3 + 2 \times 10^2 + 1 \times 10^1 + 0 \times 10^0 \\
&= 3 \times 1000 + 2 \times 100 + 1 \times 10 + 0 \times 1 \\
&= 3000 + 200 + 10 + 0 \\
&= 3210_{10}
\end{aligned}
$$

Before moving onto the binary system you may want to peak the students interest by demonstrating a math magic trick based on the binary system. It works as follows:

1) Give students the worksheet full of numbers categorized in sections A through E.
2) Ask them to pick a number between 1 and 30.
3) Ask them to find which boxes it appears in (it could be in more than one) and give the corresponding letters.
4) Use the following place value system to get a number for each letter.
   A - 16 B - 8 C - 4 D - 2 E - 1
5) Add up the numbers and you will have their number.

Example: If a student chose the number 25 it will appear in section A,B, and E
16 + 8 + 1 will give the number chosen (25).

Binary Number System

Binary Number System -- A number system having a base of two. Binary digits are digits 0 and 1.

The binary number system is much simpler than the decimal system. It is used because it is very compatible with digital electronic circuits that are either on or off. The two binary digits of 0 and 1 are used to represent this occurrence. Binary digits are sometimes called bits, which evolved from the first and last two letters of the two words *bi* nary digi*ts.*.

Place Value by Position

Digit name by position	Eights	Fours	Twos	Ones
Exponential digit value by position	2^3	2^2	2^1	2^0

We can demonstrate the value of each position(*place value*) by analyzing a sample binary number.

Example: $1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 1 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1$

$= 8 + 4 + 0 + 1$

$= 13_{10}$

This example illustrates one way to convert a binary number to its equivalent decimal value.

Chalkboard examples:

Try converting the following binary numbers to decimal.

1110_2 1111_2 1010_2

Answers: 14_{10}, 15_{10}, 10_{10}

Octal Number System

Octal Number System -- A number system, having a base of 8. Digits are 0 through 7.

Normally, data is fed to a computer in some system other than binary. This is because entering data in binary is time - consuming and prone to error. Also binary numbers are difficult to remember and express in words. The octal system is more closely related to binary than to decimal. Due to the large values of each digits position we will not name them below (digit name by position).

Place Value by Position

Exponential digit value by position	8^3	8^2	8^1	8^0

We can demonstrate the value of each position(*place value*) by analyzing a sample decimal number.

Example: $3210_8 = 3 \times 8^3 + 2 \times 8^2 + 1 \times 8^1 + 0 \times 8^0$

$$
\begin{aligned}
3210_8 &= 3 \times 8^3 + 2 \times 8^2 + 1 \times 8^1 + 0 \times 8^0 \\
&= 3 \times 512 + 2 \times 64 + 1 \times 8 + 0 \times 1 \\
&= 1536 + 128 + 8 + 0 \\
&= 1672_{10}
\end{aligned}
$$

This example illustrates one way to convert an octal number to its equivalent decimal value.

Chalkboard Examples:

Try converting the following octal numbers to decimal.

301_8 417_8 1036_8

Answers: 193_{10}, 271_{10}, 542_{10}

Hexadecimal Number System

Hexadecimal Number System -- A number system, having a base of 16. Convenient for representing 4 bit numbers. Digits are 0 through 9 and A through F.

The hexadecimal system was named from the prefix *hexa* which stands for six and *decimal* which implies ten. The first ten digits are the same as the decimal system (0-9). However, for the decimal numbers 11-15, the hexadecimal number system uses the letters A - F. Since many personal computers today use a 16-bit *assembly language*, the hexadecimal system has become very popular. The hexadecimal system is also more closely related to binary than decimal. Because of the large values for each digits position we will not name them below (digit name by position).

Place Value by Position

Exponential digit value by position	16^3	16^2	16^1	16^0

We can demonstrate the value of each position(*place value*) by analyzing a sample decimal number.

Example: 3210_{16}	=	3×16^3	+	2×16^2	+	1×16^1	+	0×16^0
	=	3 x 4096	+	2 x 256	+	1 x 16	+	0 x 1
	=	12288	+	512	+	16	+	0
	=	12816_{10}						

This example illustrates one way to convert a hexadecimal number to its equivalent decimal value.

Chalkboard Examples:

Try converting the following hexadecimal numbers to decimal.

$3C1_{16}$ $41A_{16}$ $1F06_{16}$

Answers: 961_{10}, 1050_{10}, 7942_{10}

Converting Number Systems to Binary

Digital integrated circuits handle digital information using switching circuits. These simple circuits made up of diodes, transistors, and resistors can perform the basic Boolean logic functions. Boolean logic uses the binary number system. Therefore it is important to understand how to convert number systems into binary.

Converting Decimal to Binary

Method: Successive division by two

1) Divide the decimal number by two.
2) Use the remainder from this division to fill in the ones place value.
3) Continue to divide by two on each resulting quotient.
4) Each remainder fills the next higher place value.
5) The procedure is over when you have a division of zero.

Example: 153_{10} is converted as follows.

153 / 2 = 76	remainder 1	==>	1s place
76 / 2 = 38	remainder 0	==>	2s place
38 / 2 = 19	remainder 0	==>	4s place
19 / 2 = 9	remainder 1	==>	8s place
9 / 2 = 4	remainder 1	==>	16s place
4 / 2 = 2	remainder 0	==>	32s place
2 / 2 = 1	remainder 0	==>	64s place
1 / 2 = 0	remainder 1	==>	128s place
0 / 2 = 0			

Result: $153_{10} = 10011001_2$

Chalkboard Examples:

Try converting the following decimal numbers to binary.

127_{10} 93_{10} 80_{10}

Answers: 1111111_2, 1011101_2, 1010000_2

Converting Octal to Binary

Octal is more closely related to binary than is decimal. Therefore it is important to understand how to convert octal to binary.

Method:
1) Convert each octal digit to its three-digit binary equivalent.
2) Record it from left to right starting with the first 1.

Example: 325_8 ==> 3 2 5
==> 011 010 101 (binary equivalent)

Result: 11010101_2

Chalkboard examples:

Try converting the following octal numbers to binary.

100_8 427_8 702_8

Answers: 1000000_2, 100010111_2, 111000010_2

Converting Hexadecimal to Binary

Hexadecimal is also more closely related to binary than is decimal. Again, the relationship permits easy conversion between the systems. Therefore it is important to understand how to convert hexadecimal to binary.

Method:
1) Convert each hexadecimal digit to its four-digit binary equivalent.
2) Record it from left to right starting with the first 1.

Example: $A25_{16}$ ==> A 2 5

==> 1010 0010 0101 (binary equivalent)

Result: 101000100101_2

Chalkboard examples:

Try converting the following hexadecimal numbers to binary.

$9F1_{16}$ $D03_{16}$ $52C_{16}$

Answers: 100111110001_2, 110100000011_2, 10100101100_2

At this time the students should complete the worksheet entitled Converting to Binary. After numbers in decimal, octal, and hexadecimal are converted to binary the ASCII code chart can be used to find the corresponding letter. The secret message will read: Math controls the world. For example:

> 77_{10} will be converted to 1001101_2 which corresponds to the letter M on the ASCII code chart.

Converting to Binary

	BINARY	Equivalent ASCII Letter
77_{10}		
65_{10}		
84_{10}		
72_{10}		
67_{10}		
$4F_{16}$		
$4E_{16}$		
84_{10}		
52_{16}		
$4F_{16}$		
$4C_{16}$		
123_{8}		
84_{10}		
72_{10}		
105_{8}		
127_{8}		
$4F_{16}$		
52_{16}		
$4C_{16}$		
104_{8}		

SECRET MESSAGE:

A

18 23 21
29 30
24 20 16
25 27
22 17 19
26 28

B

10 24 15
25 27
12 14 8
26 28
9 11 13
29 30

C

15 7 23
22 29
14 20 13
4 28
21 30 5
12 6

D

11 18 19
22 27
26 15 7
2 3
23 14 30
6 10

E

5 3 9
19 21
27 15 11
1 23
25 29 13
17 7

4. CODES AND SYMBOLS

ALPHABET, ASCII & MORSE CODE

ALPHABET	ASCII	MORSE CODE
A	100 0001	. –
B	100 0010	– . . .
C	100 0011	– . – .
D	100 0100	– . .
E	100 0101	.
F	100 0110	. . – .
G	100 0111	– – .
H	100 1000	. . . .
I	100 1001	. .
J	100 1010	. – – –
K	100 1011	– . –
L	100 1100	. – . .
M	100 1101	– –
N	100 1110	– .
O	100 1111	– – –
P	101 0000	. – – .
Q	101 0001	– – . –
R	101 0010	. – .
S	101 0011	. . .
T	101 0100	–
U	101 0101	. . –
V	101 0110	. . . –
W	101 0111	. – –
X	101 1000	– . . –
Y	101 1001	– . – –
Z	101 1010	– – . .
0	011 0000	– – – – –
1	011 0001	. – – – –
2	011 0010	. . – – –
3	011 0011	. . . – –
4	011 0100	. . . . –
5	011 0101	. . . . .
6	011 0110	– . . . .
7	011 0111	– – . . .
8	011 1000	– – – . .
9	011 1001	– – – – .